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DATA SMOOTHING

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MATHEMATICS RESEARCH CENTER

DATA SMOOTHING

Guang-Chang Dong*

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ABSTRACT

A new spline smoothing method is outlined. The method is modelled after the smoothing techniques used by draftsmen in ship lofting. Full details can be found in [2], [3].

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SIGNIFICANCE AND EXPLANATION

Data smoothing is probably the most difficult practical problem confronting Approximation Theory, in part because it is hard to define a satisfactory concept of "smoothness" mathematically. This report adds yet another technique to the available arsenal of methods. The technique is closely modelled on the physical processess used when ship lines are 'faired' or drafted with the aid of wooden splines. In particular, it uses something akin to weight removal and something akin to local flattening in order to achieve its goal of a curve with few inflection points yet close to the given data.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

DATA SMOOTHING

Guang-Chang Dong*

Data smoothing is an important problem in practice, such as ship lofting, car shell lines design, air-plane lines model fairing, mechanical contact line smoothing (for example, the smoothing for cam data design), etc. See, e.g., [1] for a mathematical discussion of this problem.

In this paper, a clear concept and some effective methods for data smoothing are given; this is a part of author's technical reports on mathematical ship lofting [2], [3].

1. The concept of data smoothing.

Through a set of plane data (x_i, y_i) $(1 \leq i \leq n)$ with end conditions draw the usual cubic spline

$$y = y_i + m_{i,i+1}(x-x_i) - \frac{(x-x_i)(x_{i+1}-x)}{x_{i+1}-x_i}\left[c_i(2x_{i+1}-x-x_i)+c_{i+1}(x_{i+1}+x-2x_i)\right] \tag{1}$$
$$(x_i \leq x \leq x_{i+1}, \quad i=1,2,\ldots,n)$$

where

$$m_{i,i+1} = \frac{y_{i+1}-y_i}{x_{i+1}-x_i}, \quad c_i = y_i''/6 .$$

We call $c_1,\ldots,c_n$ the coefficients of the spline.

We think of the spline as "smooth" to the extent that the curvature of the spline is varying slowly or the c_i $(i=1,2,\ldots,n)$ are varying slowly. We make this more precise in both a qualitative and a quantitative way.

Qualitative part. One requirement for a "smooth" spline is that the signature of c_i should change slowly or the number of inflection points R of the spline curve should not be large. If the original R is large, we must change the data by a little amount so

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as to reduce R, and this is a part of the smoothing procedure. But what is the suitable final value for R? There is no absolute standard. The suitable final value for R is dependent on our requirement, i.e., we should give the required value for R before smoothing. Auxiliary before smoothing we also give the qualitative end requirements: We require that the spline on left end is convex (given $D_1=1$) or concave (given $D_1=-1$). The same for right end (given $D_2=1$ or $D_2=-1$).

Summing up, the three numbers D_1, D_2, R are given at the outset.

Quantitative part. Another requirement for a "smooth" spline derives from an analysis of the force from a duck acting on a wooden spline. The exterior force on node (x_i, y_i) in the y direction equivalent to the duck holding the wooden spline fixed is approximately equal to

$$P_i = 6EIe_i := 6EI\left(\frac{c_{i+1}-c_i}{x_{i+1}-x_i} - \frac{c_i-c_{i-1}}{x_i-x_{i-1}}\right) \tag{2}$$

where EI is the bending rigidity of the wooden spline. The ideal smooth case for a spline would be for the exterior force acting on consecutive nodes to have the same signature, or

$$e_i e_{i+1} > 0 \quad (i=2,3,\ldots,n-2) \quad . \tag{3}$$

Express (3) geometrically. The $c(x) := y''(x)/6$ curve consists of some convex and concave polygons. Then (3) demands that the connecting point (x_i, c_i) of neighboring convex and concave polygons have the property that (x_{i-1}, c_{i-1}), (x_i, c_i), (x_{i+1}, c_{i+1}) lie on a straight line.

We do not attempt to take the ideal case as the goal for smoothing, otherwise one would need to change the data too much. Obviously, keeping change of data to a minimum is also a requirement for data smoothing. The non smooth case is

$$e_j e_{j+1} < 0 \tag{4}$$

with both e_j and e_{j+1} large in absolute value. More accurately, the amount of spring-back when we release a single duck on (x_j, y_j) or (x_{j+1}, y_{j+1}) are both large. Hence a part of our smoothing procedure is to reduce the quantity of spring-back to a certain amount when (4) is true.

Before turning to describe the methods of smoothing, some general remarks should be given.

1. In our methods of smoothing the two end data are always kept fixed.

2. We use just two types of end conditions. For the left end, we give y_1' (fixed end) or else give no slope condition (free end). In the free end case, we always add the condition $c_1 = c_2$, i.e., we require that the first section of the spline is a parabolic segment. The right end is treated analogously.

3. In addition to keeping the end data fixed, we can fix a few other data also.

4. If the spline drawn through the given data set becomes $|y'(x)| > 1$ on some segment, then the principal part of the exterior force is not in the y direction expressed by (2), and the effect of our method comes into doubt. For this reason, we restrict our method to data which satisfy the condition $|y'(x)| \leq 1$ $(x_1 \leq x \leq x_n)$.

5. We can treat nearby pairs of points; i.e., pairs of points (x_j, y_j), (x_{j+1}, y_{j+1}) with $x_{j+1} - x_j$ less than a quantity given at our disposal.

2. The Resilience Method.

Given a set of data (x_i, y_i) $(1 \leq i \leq n)$ with end conditions, draw a spline with coefficients $c_1, \ldots, c_n$. Change y_j to $y_j + \delta y_j$ $(2 \leq j \leq n-1)$, draw another spline with coefficients $\tilde{c}_1, \ldots, \tilde{c}_n$. We have

$$\tilde{c}_i = c_i + c_i^{(j)} \delta y_j \quad (1 \leq i \leq n) \tag{5}$$

where $c_i^{(j)}$ $(1 \leq i \leq n)$ are the coefficients of the j-th unit spline, i.e., the spline for $y_j = 1$, $y_i = 0$ $(i \neq j)$ and homogeneous end conditions. We can prove that

$$(-1)^{j-i} c_i^{(j)} < 0 \quad (\alpha \leq i \leq \beta) \tag{6}$$

where $\alpha=1$, $\beta=n$ for fixed end and $\alpha=2$, $\beta=n-1$ for free end. Moreover

$$|c_i^{(j)}| \text{ diminishes monotonely and quickly (at least as fast as } 2^{-|i-j|}) \quad \text{for } |i-j| \geq 2 \ . \tag{7}$$

When we release a duck acting at (x_j, y_j) on a wooden spline, i.e., we set

$$\tilde{P}_j = 6EI\tilde{e}_j = 0$$

then, by (5), we have

$$\delta y_j = -e_j/e_j^{(j)} \tag{8}$$

for the resulting change δy_j. From (6) we have $e_j^{(j)} > 0$, so that by (8) we have $\delta y_j e_j < 0$. This is the source of the name "Resilience Method".

Smoothing from $e_j e_{j+1} < 0$ to $\tilde{e}_j \tilde{e}_{j+1} = 0$ by only releasing one duck may make the datum change by a large amount, so we consider the change of two data δy_j and δy_{j+1}. We have

$$\tilde{c}_i = c_i + c_i^{(j)}\delta y_j + c_i^{(j+1)}\delta y_{j+1} \quad (1 \leq i \leq n) \ .$$

Let $\delta y_j = -\delta y_{j+1}$ and take the minimum value of $|\delta y_j|$ such that $\tilde{e}_j \tilde{e}_{j+1} = 0$. It is easy to obtain the formula

$$\delta y_j = -\delta y_{j+1} = \sigma_{j,j+1} := (-\text{sign } e_j) \min\left\{\frac{|e_j|}{e_j^{(j)}-e_j^{(j+1)}}, \frac{|e_{j+1}|}{e_{j+1}^{(j+1)}-e_{j+1}^{(j)}}\right\} \ . \tag{9}$$

In the actual smoothing process, we do not need to make the resilient quantity disappear, but only reduce it. We distinguish three cases as follows:

1. When

$$e_{j-1}e_j > 0,\ e_j e_{j+1} < 0,\ e_{j+1}e_{j+2} > 0$$

we take

$$\delta y_j = -\delta y_{j+1} = 0.8\sigma_{j,j+1} \qquad (10)$$

2. When $k > j+1$ and

$$e_{j-1}e_j > 0,\ e_j e_{j+1} < 0, \ldots, e_{k-1}e_k < 0,\ e_k e_{k+1} > 0$$

we take

$$\begin{aligned} &\delta y_j = 0.4\sigma_{j,j+1},\ \delta y_{j+1} = 0.4(-\sigma_{j,j+1}+\sigma_{j+1,j+2}), \ldots, \\ &\delta y_{k-1} = 0.4(-\sigma_{k-2,k-1}+\sigma_{k-1,k}),\ \delta y_k = -0.4\sigma_{k-1,k} \end{aligned} \qquad (11)$$

3. When there is an inflection point in (x_j, x_{j+1}), and in (x_{j-1}, x_{j+2}) there is only this one inflection point, i.e., when

$$c_j c_{j+1} < 0,\ c_{j-1}c_j > 0,\ c_{j+1}c_{j+2} > 0 \ ,$$

in this case we replace $\sigma_{j,j+1}$ by 0 in (10) or (11).

If we meet a fixed point, then at this point we simply do not make any adjustment.

This process is called one-time relaxation. How many times shall we carry out our process? When we find that $\max_j|\sigma_{j,j+1}|$ is less than an assigned quantity ε of our choice, we carry out the process one more time and then stop. For example, for ship lofting we always take ε=3 mm.

Before we apply the Resilience Method, we first treat all 'nearby' points as follows. Let $\delta y_j = -\delta y_{j+1} = \sigma$ so as to achieve $\tilde{e}_j = \tilde{e}_{j+1}$; it is easy to see in this case

$$\sigma = \frac{e_{j+1} - e_j}{e_j^{(j)} - e_j^{(j+1)} + e_{j+1}^{(j)} - e_{j+1}^{(j+1)}} .$$

3. The Rule Modelling Method.

First let us give an example.

When we find a number $S = 1$ or -1 such that

$$Sc_{j-1} > 0,\ Sc_j < 0,\ Sc_{j+1} < 0,\ Sc_{j+2} > 0\ (1 < j < n-2) \tag{12}$$

we try to alter y_j, y_{j+1} such that

$$Sc(x) > 0\ (x_{j-1} < x < x_{j+2}) \tag{13}$$

We have

$$\begin{aligned} \tilde{c}_j &= c_j + c_j^{(j)} \delta y_j + c^{(j+1)} \delta y_{j+1} \\ \tilde{c}_{j+1} &= c_{j+1} + c_{j+1}^{(j)} \delta y_j + c_{j+1}^{(j+1)} \delta y_{j+1} \end{aligned} . \tag{14}$$

First step. If $\tilde{c}_j = \tilde{c}_{j+1} = 0$, solve for δy_j, δy_{j+1} and adjust. If the signature of $\tilde{c}_{j-1}$, $\tilde{c}_{j+2}$ is the same as S, then go to second step. Otherwise we have met an unusual situation and discuss it later.

Second step. When

$$Sc_{j-1} > 0,\ c_j = c_{j+1} = 0,\ Sc_{j+2} > 0 \quad , \tag{15}$$

let

$$\tilde{c}_j = \tilde{c}_{j+1} = S \min\{|c_{j-1}|, |c_{j+2}|\}/5 \tag{16}$$

and solve for δy_j, δy_{j+1} in (14), and adjust. We get the final result (13).

In the first step $\tilde{c}_j = \tilde{c}_{j+1} = 0$, i.e., $\tilde{y}(x)$ has straight line segment in $[x_j, x_{j+1}]$. This corresponds to flattening a wooden spline by pushing a ruler against it so as to remove the $Sc(x) < 0$ part on the spline curve. This is the source of the name "Rule Modelling Method". Our final result is $\tilde{c}_j = \tilde{c}_{j+1}$ expressed by (16), i.e., $y(x)$ has parabolic segment in $[x_j, x_{j+1}]$.

We now give a detailed description of the Rule Modelling Method.

Given the three numbers D_1, D_2, R. Restrict D_1 to only take the values 1, -1, or 0. For the left end we require that

$$\begin{array}{ll} D_1 c_1 > 0 & (\text{when } D_1 \neq 0) \\ c_1 c_2 > 0 & (\text{when } D_1 = 0) \end{array} \tag{17}$$

similarly for the right end. And we require that the total number of points of inflection of the spline does not exceed R.

Suppose the above three requirements are not all satisfied. We start by finding the interval $[x_j, x_k]$ for adjustment and a number S ($S = 1$ or -1) satisfying

$$Sc_j < 0,\ Sc_{j+1} < 0, \ldots, Sc_k < 0,\ Sc_{j-1} > 0 \ (\text{when } j \neq 1),\ Sc_{k+1} > 0 \ (\text{when } k \neq n) \ . \tag{18}$$

If the left end requirement (17) is not satisfied, let $x_j = x_1$. Let $S = D_1$ or $S = \text{sign } c_2$ according to $D_1 \neq 0$ or else. Determine x_k by the conditions $Sc_h < 0$ $(1 < h < k)$, $Sc_{k+1} > 0$.

If the right end condition is not satisfied, find S and $[x_j, x_k]$ similarly.

If end conditions are satisfied, and condition on inflection points is not, then firstly we try to find j in $[2, n-1]$ such that $c_{j-1}c_j < 0$, and $c_{j-1}c_{j+1} > 0$ and, if such j exists, let $S = \text{sign } c_{j-1}$ and $x_k = x_j$. Otherwise try to find j in

$[2, n-2]$ such that $c_{j-1}c_j < 0$, $c_{j-1}c_{j+1} < 0$, $c_{j-1}c_{j+2} > 0$, and, if such j exists, let $S = \text{sign } c_{j-1}$ and $x_k = x_{j+1}$, etc.

Thus we find (x_j, x_k) and S satisfying (18). The cases $k=j$, $k=j+1$, $k > j+1$ are called incorrect small, large, and extremely large wave.

We are restricted to at most one fixed point of the spline in the interval of adjustment $[x_j, x_k]$. But there is an exceptional case, viz, $j=1$, the signature of c is not correct, we need to adjust and y_1' is given. In this case, we cannot admit any fixed point in $[x_j, x_k]$; similar for the right end case.

Removing an incorrect extremely large wave can be reduced to the case of removing an incorrect large wave by deleting the nodes between x_j, x_k and forming a new spline with $n-k+j+1$ nodes, adjust and then recover the spline with n nodes by spline interpolation. Thus we only give the formulas for removing incorrect small and large waves in the middle or near the left end, because near the right end situation is similar.

Formula for removing incorrect small wave:

When $1 < j=k < n$, we use the formula

$$\tilde{c}_j = c_j + \delta y_{j-1} c_j^{(j-1)} + \delta y_j c_j^{(j)} + \delta y_{j+1} c_j^{(j+1)} := c_j + \sigma\left(\alpha c_j^{(j-1)} + \beta c_j^{(j)} + \gamma c_j^{(j+1)}\right) \tag{19}$$

where in general we have

$$\alpha = 1, \ \beta = -1.5, \ \gamma = 1 \tag{20}$$

i.e., when (x_{j-1}, y_{j-1}), $(_j, y_j)$, (x_{j+1}, y_{j+1}) are not fixed points, then (20) is true. If (x_{j-1}, y_{j-1}) is a fixed point, let $\alpha = 0$, etc.

First step. Let $\tilde{c}_j = 0$ in (19) to find σ, then adjust. If $S\tilde{c}_{j-1} > 0$, $S\tilde{c}_{j+1} > 0$ are true, go to second step, otherwise we have met an unusual situation and discuss it later.

<u>Second step.</u> From $Sc_{j-1} > 0$, $c_j = 0$, $Sc_{j+1} > 0$ we take

$$\tilde{c}_j = S\min(|c_{j-1}|, |c_{j+1}|)/5 \tag{21}$$

and determine σ by (19), then adjust. We get the final result

$$Sc(x) > 0 \quad (x_{j-1} \leq x \leq x_{j+1}) \quad .$$

When $j = k = 1$, take $\alpha = \beta = 0$, $\gamma = 1$ in (19). Instead of (21), we take

$$\tilde{c}_1 = c_2/8 \quad . \tag{22}$$

<u>Formulas for removing incorrect large wave:</u>

This is the case (12). If there is no fixed point in $[x_j, x_{j+1}]$, adjust as in (14), (15), (16).

If one of (x_j, y_j), (x_{j+1}, y_{j+1}) is a fixed point, consider the formula

$$\tilde{c}_i = c_i + c_i^{(j-1)}\delta y_{j-1} + c_i^{(j)}\delta y_j + c_i^{(j+1)}\delta y_{j+1} + c_i^{(j+2)}\delta y_{j+2} \quad (1 \leq i \leq n) \tag{23}$$

taking

$$\delta y_{j-1} = \sigma\alpha, \; \delta y_{j+2} = \sigma\beta, \; \delta y_j = \tau\gamma, \; \delta y_{j+1} = \tau\delta \quad . \tag{24}$$

Take $\gamma = 1$, $\delta = 0$ when (x_{j+1}, y_{j+1}) is fixed. Take $\gamma = 0$, $\delta = 1$ when (x_j, y_j) is fixed. Take $\alpha = 0$ if (x_{j-1}, y_{j-1}) is a fixed point or $j = 2$, or x_{j+1} is an extended end point (see below) which we marked before, otherwise take $\alpha = 1$. Take $\beta = 0$ if

(x_{j+2}, y_{j+2}) is a fixed point or $j = n-2$ or x_j is an extended end point, otherwise take $\beta = 1$.

Solve for σ, τ by substituting (23) - (24) into $\tilde{c}_j = \tilde{c}_{j+1} = 0$, then adjust. This is the first step. Use (16) for the second step adjustment.

If there is a fixed point (x^*, y^*) satisfying $x_j < x^* < x_{j+1}$, and $k > j+1$ and we delete the data points between (x_j, x_k) on the spline, then the fixed point would be lost, too. In this case the modeling rule cannot move freely and can only change the slope through the point (x^*, y^*). Following this idea, we give the adjusting steps as follows:

First step. Take $y^*+y'(x^*)(x_j-x^*)$, $y^*+y'(x^*)(x_{j+1})$ instead of y_j, y_{j+1}, draw spline. In general, this spline does not pass through the point (x^*, y^*). Take

$$\gamma = \frac{x_j-x^*}{x_{j+1}-x_j}, \quad \delta = \frac{x_{j+1}-x^*}{x_{j+1}-x_j}$$

in (23), take α, β as before. Solve for σ, τ by substituting (23) - (24) in $\tilde{c}_j = \tilde{c}_{j+1} = 0$, adjust, draw the spline. This spline contains a line segment in $x_j \leq x \leq x_{j+1}$, and (x^*, y^*) lies on this line segment.

Second step. Denote the number in formula (16) by λ, add $3\lambda(x^*-x_j)(x_{j+1}-x^*)$ to y_j, y_{j+1}, draw a spline. The final spline passes through the point (x^*, y^*) and contains a parabolic segment with $y'' = 6\lambda$ in $x_j \leq x \leq x_{j+1}$ and satisfying (13).

If $j = 1$, there is an incorrect large wave at the left end. If there is no fixed point in $(x_1, x_2]$ and y_1' is not fixed, replace (14) by

$$\tilde{c}_i = c_i + c_i^{(1)}\delta y_1' + c_i^{(2)}\delta y_2 \quad (1 \leq i \leq n) \tag{25}$$

where $c_i^{(1)}$ are the coefficients of the unit spline given by $y_i = 0$ $(1 \leq i \leq n)$, $y_1' = 1$, and homogeneous right end condition. Replace (16) by

$$\tilde{c}_1 = \tilde{c}_2 = c_3/8 \quad . \tag{26}$$

Here we already assume that $Sc_3 > 0$, otherwise we shoul. delete nodes to treat the incorrect extremely large end wave.

Suppose there is a fixed point (x^*,y^*) in $x_1 < x^* \leq x_2$ or y_1' is fixed. In these cases the modeling rule is fixed, it is the straight line through (x_1,y_1), (x^*,y^*) in the former case and through (x_1,y_1) with slope y_1' in the latter case. Following this idea we give the steps for adjustment.

First step. Replace y_1' and y_2 by $\frac{y^*-y_1}{x^*-x_1}$ and $y_1 + \frac{y^*-y_1}{x^*-x_1}(x_2-x_1)$ in the former and replace y_2 by $y_1+y_1'(x_2-x_1)$ in the latter case. Draw the spline, in general this spline does not pass through the point (x^*,y^*). Replace (14) by

$$\tilde{c}_i = c_i+c_i^{(3)}\delta y_3 \qquad (1 \leq i \leq n) \quad . \tag{27}$$

Let $\tilde{c}_2=0$ in (27), adjust. It is easy to prove that $\tilde{c}_1 = 0$. Hence the spline contains a straight line segment in $[x_1,x_2]$ passing through the point (x^*,y^*).

Second step. Add $3\lambda(x_1-x^*)$, $3\lambda(x_2-x_1)(x_2-x^*)$ (where $\lambda = c_3/8$) to y_1' and y_2, draw spline, Let $\tilde{c}_2 = c_3/8$ in (26), adjust. Then we can prove that $\tilde{c}_1 = c_3/8$ and satisfying $Sc(x) > 0$ in $x_1 \leq x \leq x_3$.

Thus, all the cases for adjustment are stated.

The rule for treating an unusual situation is as follows:

When we do the first step for adjustment, if at least one of $S\tilde{c}_{j-1} \leq 0$, $S\tilde{c}_{j+h} \leq 0$ $(h = 1$ or 2 according to $k = j$ or $k = j+1)$ happens, we distinguish four cases.

First case. Both $S\tilde{c}_{j-1} \leq 0$ and $S\tilde{c}_{j+h} \leq 0$ are true.

In this case simply turn to second step, only need a changed signature in the right hand side of (21) or (16).

Second case. One of $S\tilde{c}_{j-1} < 0$, $S\tilde{c}_{j+h} < 0$ is true, with $\tilde{c}_{j-1}\tilde{c}_{j-2} > 0$, $\tilde{c}_{j+h}\tilde{c}_{j+h+1} > 0$.

In this case simply turn to second step with the following change. In (16), replace S by sign c_α when

$$\min\{|c_{j-1}|, |c_{j+1}|\} = |c_\alpha| \quad ; \tag{28}$$

similar for (21).

Third case. One of $S\tilde{c}_{j-1} < 0$, $S\tilde{c}_{j+h} < 0$ is true, with $S\tilde{c}_{j-2} > 0$, $S\tilde{c}_{j+h+1} > 0$.

In this case we need to extend the adjusting interval. After a new first step no matter what the signature $S\tilde{c}_{j-1}$, $S\tilde{c}_{j+h}$ are, we never extend adjusting interval again except when $j = 1$ and $S = D_1$. Turn to second step, with S replaced by sign c_α, where c_α is defined by (28).

Fourth case. When $S\tilde{c}_{j-1} < 0$ and (x_j,y_j), (x_{j-1},y_{j-1}) are nearby points or $S\tilde{c}_{j+h} < 0$ and (x_{j+h-1},y_{j+h-1}), (x_{j+h},y_{j+h}) are nearby points, in any of these cases we always extend the adjusting interval.

Remark. In the above cases, when $j = 1$, and so $\tilde{c}_{j-1}$ does not exist, when $j = 2$, and so $\tilde{c}_{j-2}$ does not exist, etc, we use the convention that then the corresponding requirements $S\tilde{c}_{j-1} < 0$, $\tilde{c}_{j-1}\tilde{c}_{j-2} < 0$, ... are not considered.

It only happens when we wish to remove an incorrect end wave that we need to extend the adjusting interval again and again until the incorrect wave has been removed, or else we arrive at the extreme case $[x_j,x_k] = [x_1,x_n]$. The extreme case happens when one of the given numbers D_1, D_2, R is not reasonable.

In principle we can admit in $[x_j,x_k]$ more than one fixed point, in this case the modeling rule is fixed, but in our practice we have not met it and we do not want to make our program more complex, so that this case has not been considered. Only a similar case happened in practice and is considered by us: There exists an incorrect left end small wave and both y_1' and (x_2,y_2) are fixed, and in this case we adjust by simply adding $-1.1c_1/c_1^{(3)}$ to y_3, similar for the right end.

We have finished our statement of removing incorrect wave. If the three requirements D_1, D_2, R are still not satisfied, then we adjust cyclically.

With two or more large (and extremely large) waves existing and $R \leq 2$, we cannot determine from D_1, D_2, R which one is the incorrect large (or extreme large) wave. When this situation happens (in fact it happens rarely), we print $c_i(i=1,\ldots,n)$ and let a man judge, he would specify the interval $[x_j,x_k]$ of incorrect wave, and then do the business of removing incorrect wave by computer.

Removing incorrect large or extreme large wave may produce small incorrect wave. Removing incorrect internal wave may produce incorrect end wave. But the probability of this happening is small by (7). So that when we finish the process for removing incorrect waves once, and the three requirements for D_1, D_2, R are still not satisfied, we should repeat the entire process again and again, until the three requirements are satisfied. If after repeating the whole process n times the three requirements still are not satisfied then we stop. In effect, we then judge the three given numbers D_1, D_2, R to be inconsistent with the given data.

4. Local adjustment for smoothing.

After the qualitative requirement on the spline is already satisfied and should we then be forbidden to use the whole line Resilient Method, in this situation we can use some supplementary methods of local adjustment for smoothing. They are as follows:

A. Adjustment of local part of spline which is too flat.

Assign a constant K greater than 1, usually we take

$$K \in \{5, 3, 2.5, 2\} \quad . \tag{29}$$

<u>Case 1.</u> c_{j-1}, c_j, c_{j+1} have the same signature and

$$K|c_j| < \min\{|c_{j-1}|, |c_{j+1}|\} \quad . \tag{30}$$

<u>Case 2.</u> $c_{j-1}, c_j, c_{j+1}, c_{j+2}$ have the same signature and

$$K|c_j| < |c_{j-1}|, \quad K|c_{j+1}| < |c_{j+2}| \quad . \tag{31}$$

These two cases are called <u>locally too flat.</u>

For case 1, take $\delta y_{j-1}, \delta y_j, \delta y_{j+1}$ and α, β, γ as in (19), (20) and require that

$$K|\tilde{c}_j| = \min\{|\tilde{c}_{j-1}| , |\tilde{c}_{j+1}|\} \quad . \tag{32}$$

We can find σ and adjust.

For case 2, assume that there can be at most one fixed point between (x_{j-1}, y_{j-1}), (x_j, y_j) and similarly for $(x_{j+1}, y_{j+1}), (x_{j+2}, y_{j+2})$. Take

$$\sigma y_{j-1} = \alpha\sigma, \ \delta y_j = \beta\sigma, \ \delta y_{j+1} = \gamma\tau, \ \delta y_{j+2} = \delta\tau \tag{33}$$

in (19), and in general let

$$\alpha = \delta = 2, \quad \beta = \gamma = -1 \quad . \tag{34}$$

But when (x_{j-1}, y_{j-1}) is a fixed point let $\alpha = 0$, similar for β, γ, δ. We require that

$$K\tilde{c}_j = \tilde{c}_{j-1}, \quad K\tilde{c}_{j+1} = \tilde{c}_{j+2} \quad . \tag{35}$$

We can find σ, τ and adjust.

B. Adjusting of local part of spline which is too bent.

Case 1. c_h $(j-2<h<j+2)$ have the same signature and

$$|c_j| > K \max\{|c_{j-1}|, |c_{j+1}|\} , \tag{36}$$

K as in (29).

Case 2. c_j $(j-2<h<j+3)$ have the same signature and

$$|c_j| > K|c_{j-1}| , |c_{j+1}| > K|c_{j+2}| . \tag{37}$$

These two cases are called locally too bent. Notice that in some cases locally too bent is reasonable, so that we then would use the Resilience Method to adjust in such a way that the reasonable bent is retained.

We adjust locally by Resilience Method in the interval $[x_{j-1},x_{j+h}]$ once, where $h = 1$ or 2 for case 1 or 2 respectively.

C. Adjustment of local part of spline which has one directional large curvature jump.

This occurs when c_{j-1}, c_j, c_{j+1}, c_{j+2} have the same sigature and

$$|c_j| > K|c_{j+1}|, c_j^2 > Kc_{j-1}c_{j+1}, c_jc_{j+2} > Kc_{j+1}^2 , \tag{38}$$

K as in (29). Take

$$\delta y_j = \alpha\sigma, \delta y_{j+1} = \beta\sigma \tag{39}$$

in (14) and in general let $\alpha = 2$, $\beta = -1$, but when (x_j,y_j) is a fixed point let $\alpha = 0$, similar for β. Then determine σ by the condition

$$\tilde{c}_j = K\tilde{c}_{j+1}$$

and adjust.

Interchanging c_j and c_{j+1}, c_{j-1} and c_{j+2}, we obtain another case needing adjustment.

After local adjustment, the three qualitative requirements may still not be satisfied, so that we must do qualitative adjustment again.

5. A few words on programming.

The effective program for data smoothing ought to have many capabilities such as:

1. Can rotate axis, such that the condition $|y'(x)| < 1$ on the whole spline is satisfied.
2. Can deal with nearby points or not, as wanted.
3. Can do any of the five methods of smoothing separately: the Resilience Method R, the Rule Modelling Method M, the method A, B, C of 4.

Can do the above methods in some suitable order such as RM, MAM, MBM, MCM, MABCM, RMABCM.

It is also required that using the program we can obtain the smoothing result quickly. This requires that the coefficients of the spline be calculated quickly, e.g., by some local method. For this, we derived a series of formulas, they are written down in [3].

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A new spline smoothing method is outlined. The method is modelled after the smoothing techniques used by draftsmen in ship lofting. Full details can be found in [2], [3].

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